Improved Surface Integral Equation-Based Formulation for Characteristic Modes of Composite Metallic-Dielectric Objects

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I. INTRODUCTION

Theory of characteristic modes (TCM) has attracted much interest in EM community because it affords a convenient approach to determine the resonant behavior and get modal solutions of arbitrarily-shaped objects, without considering specific excitation sources. However, it is challenging to calculate the characteristic modes (CMs) of complex structures, especially for those structures including dielectric bodies. For such structures, the volume integral equation (VIE)-based CM formulation is robust but very time-consuming. The surface integral equation (SIE)based CM formulations require that one of the equivalent electric and magnetic currents on dielectric surfaces must be eliminated to suppress spurious modes. Existing SIE-based CM formulations use an improper way to conduct the elimination by assuming that incident fields are zero, which is not natural. In this paper, we propose a more natural way to develop two newly improved SIE-based CM formulations. Numerical results show that the proposed formulations can provide accurate modal solutions to composite metallic-dielectric problems.

II. DERIVATION OF FORMULATIONS

Considering a composite metallic-dielectric object shown in Fig. 1a, the object can be decomposed into separated structures by contact-region modeling technique, as displayed in Fig. 1b. Applying the surface equivalence principle to the problem of Fig. 1b, it can be decomposed into two sub-problems, as shown in Fig. 1c and 1d.

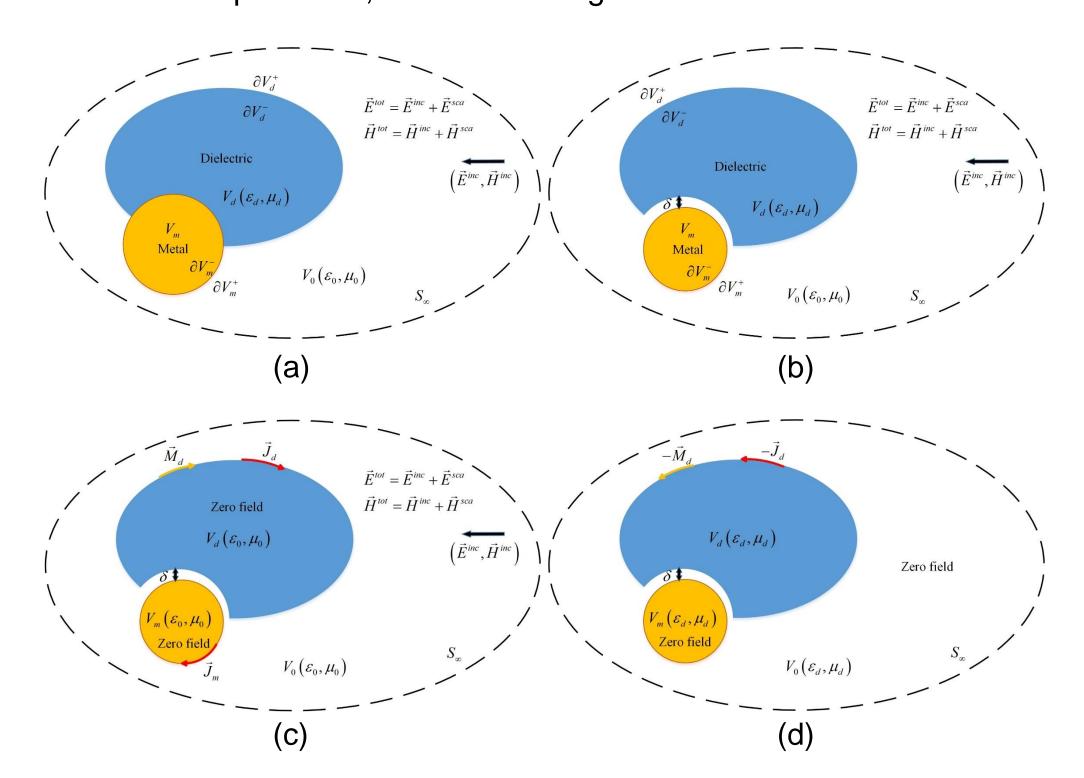


Fig. 1. Configuration of a composite metallic-dielectric object and its equivalent problems. (a) Original configuration. (b) Decomposed by contact-region modeling technique. (c) External equivalent problem. (d) Internal equivalent problem.

In the external equivalent problem, it is assumed that the fields inside the dielectric and metallic regions both are zero, while the fields in the background media remain unchanged. According to the surface equivalence principle, we have

$$\begin{cases} -j\omega\mu_{0}\mathbf{L}_{\mathbf{dm}}^{\mathbf{0}}\left(\vec{J}_{m}\right) - j\omega\mu_{0}\mathbf{L}_{\mathbf{dd}}^{\mathbf{0}}\left(\vec{J}_{d}\right) - \mathbf{K}_{\mathbf{dd}}^{\mathbf{0}+}\left(\vec{M}_{d}\right) + \vec{E}^{inc} = 0 \\ \mathbf{K}_{\mathbf{dm}}^{\mathbf{0}-}\left(\vec{J}_{m}\right) + \mathbf{K}_{\mathbf{dd}}^{\mathbf{0}+}\left(\vec{J}_{d}\right) - j\omega\varepsilon_{0}\mathbf{L}_{\mathbf{dd}}^{\mathbf{0}}\left(\vec{M}_{d}\right) + \vec{H}^{inc} = 0 \\ -j\omega\mu_{0}\mathbf{L}_{\mathbf{mm}}^{\mathbf{0}}\left(\vec{J}_{m}\right) - j\omega\mu_{0}\mathbf{L}_{\mathbf{md}}^{\mathbf{0}}\left(\vec{J}_{d}\right) - \mathbf{K}_{\mathbf{md}}^{\mathbf{0}-}\left(\vec{M}_{d}\right) + \vec{E}^{inc} = 0 \end{cases}$$

In the internal equivalent problem, it is assumed that the fields inside the dielectric region remain unchanged, while the fields outside it are zero. Based on the boundary conditions on the dielectric surface, we have

$$\begin{cases} j\omega\mu_{d}\mathbf{L}_{\mathbf{dd}}^{\mathbf{d}}\left(\vec{J}_{d}\right) + \mathbf{K}_{\mathbf{dd}}^{\mathbf{d}-}\left(\vec{M}_{d}\right) = 0 \\ -\mathbf{K}_{\mathbf{dd}}^{\mathbf{d}-}\left(\vec{J}_{d}\right) + j\omega\varepsilon_{d}\mathbf{L}_{\mathbf{dd}}^{\mathbf{d}}\left(\vec{M}_{d}\right) = 0 \end{cases}$$

$$\begin{cases} \vec{M}_{d} = \mathbf{T} \cdot \vec{J}_{d}; \quad \vec{J}_{d} = \mathbf{T}^{-1} \cdot \vec{M}_{d} \\ \mathbf{T} = -\left(j\omega\mu_{d}\mathbf{L}_{\mathbf{dd}}^{\mathbf{d}} - \mathbf{K}_{\mathbf{dd}}^{\mathbf{d}-}\right) \cdot \left(j\omega\varepsilon_{d}\mathbf{L}_{\mathbf{dd}}^{\mathbf{d}} + \mathbf{K}_{\mathbf{dd}}^{\mathbf{d}-}\right)^{-1} \end{cases}$$

Above equations demonstrate that the equivalent electric and magnetic currents on the dielectric surfaces satisfy <u>inherent independent</u> <u>relationship that is independent on incident fields</u>. In the derive of CM formulations, the dependent relationship should be applied to suppress spurious modes. The most commonly used approach is to eliminate one of the electric and magnetic currents by using the dependent relationship. Combining the equations of external and internal problems, we have

$$\begin{bmatrix} j\omega\mu_{0}\mathbf{L}_{\mathbf{mm}}^{\mathbf{0}} & j\omega\mu_{0}\mathbf{L}_{\mathbf{md}}^{\mathbf{0}} & \mathbf{K}_{\mathbf{md}}^{\mathbf{0}} \\ j\omega\mu_{0}\mathbf{L}_{\mathbf{dm}}^{\mathbf{0}} & j\omega\left(\mu_{0}\mathbf{L}_{\mathbf{dd}}^{\mathbf{0}} + \mu_{d}\mathbf{L}_{\mathbf{dd}}^{\mathbf{d}}\right) & \mathbf{K}_{\mathbf{dd}}^{\mathbf{0}+} + \mathbf{K}_{\mathbf{dd}}^{\mathbf{d}-} \\ -\mathbf{K}_{\mathbf{dm}}^{\mathbf{0}-} & -\left(\mathbf{K}_{\mathbf{dd}}^{\mathbf{0}+} + \mathbf{K}_{\mathbf{dd}}^{\mathbf{d}-}\right) & j\omega\left(\varepsilon_{0}\mathbf{L}_{\mathbf{dd}}^{\mathbf{0}} + \varepsilon_{d}\mathbf{L}_{\mathbf{dd}}^{\mathbf{d}}\right) \end{bmatrix} \cdot \begin{bmatrix} \vec{J}_{m} \\ \vec{J}_{d} \\ \vec{M}_{d} \end{bmatrix} = \begin{bmatrix} \vec{E}^{inc} \\ \vec{E}^{inc} \\ \vec{H}^{inc} \end{bmatrix} \Leftrightarrow \mathbf{Z} \cdot f = g$$

Eliminating one of the electric and magnetic currents on the dielectric surface by using the dependent relationship, we have

$$\mathbf{Z}_{\mathbf{J}} \cdot \begin{bmatrix} \vec{J}_{m} \\ \vec{J}_{d} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{T} \end{bmatrix}^{H} \cdot g = g_{J}$$

$$\mathbf{Z}_{\mathbf{M}} \cdot \begin{bmatrix} \vec{J}_{m} \\ \vec{M}_{d} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{H} \cdot g = g_{M}$$

$$\mathbf{Z}_{\mathbf{J}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{H} \cdot \mathbf{Z} \cdot \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

$$\mathbf{Z}_{\mathbf{M}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{H} \cdot \mathbf{Z} \cdot \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

Finally, CMs are solved by following generalized eigenvalue equations

$$\mathbf{X_{J}} \cdot \begin{bmatrix} \vec{J}_{m} \\ \vec{J}_{d} \end{bmatrix}_{n} = \lambda_{n} \mathbf{R_{J}} \cdot \begin{bmatrix} \vec{J}_{m} \\ \vec{J}_{d} \end{bmatrix}_{n} \qquad \text{or} \qquad \mathbf{X_{M}} \cdot \begin{bmatrix} \vec{J}_{m} \\ \vec{M}_{d} \end{bmatrix}_{n} = \lambda_{n} \mathbf{R_{M}} \cdot \begin{bmatrix} \vec{J}_{m} \\ \vec{M}_{d} \end{bmatrix}_{n}$$
 in which

 $\begin{cases} \mathbf{Z_{J}} = \mathbf{R_{J}} + j\mathbf{X_{J}} \\ \mathbf{R_{J}} = \frac{\mathbf{Z_{J}} + \mathbf{Z_{J}}^{H}}{2} \\ \mathbf{X_{J}} = \frac{\mathbf{Z_{J}} - \mathbf{Z_{J}}^{H}}{2 i} \end{cases}$

 $\begin{cases} \mathbf{Z_M} = \mathbf{R_M} + j\mathbf{X_M} \\ \mathbf{R_M} = \frac{\mathbf{Z_M} + \mathbf{Z_M}^H}{2} \\ \mathbf{X_M} = \frac{\mathbf{Z_M} - \mathbf{Z_M}^H}{2j} \end{cases}$

III. RESULTS AND CONCLUSION

Comparing the modal results of a spherical conductor with three-layer dielectric coatings solved from the VIE-based method (FEKO) and the proposed methods shows that they provide similar modal solutions, as shown in Fig. 3 and 4. Since the VIE-based method is robust, it can be concluded that the proposed formulations are robust, too.

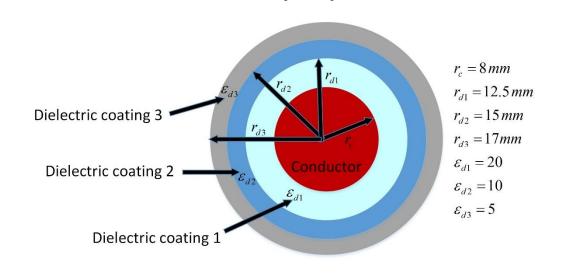


Fig. 2. Sectional view of a spherical conductor with three-layer dielectric coatings.

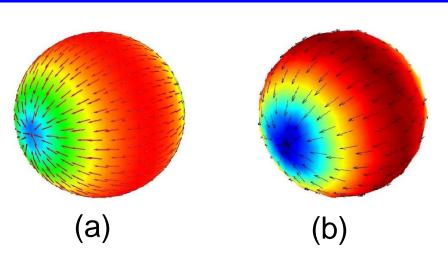


Fig. 4. Modal currents on the metallic surface at the lowest resonant frequency. (a) VIE-based method. (b) Proposed method.

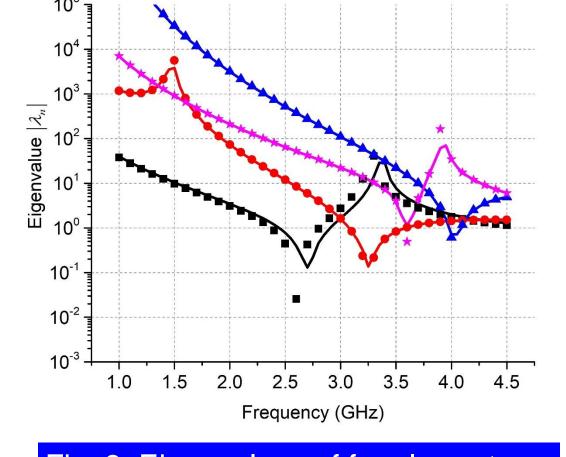


Fig. 3. Eigenvalues of four lowest modes obtained from the VIE-based method (solid lines) and the proposed method (symbols).







