



High Order Interpolation Error Analysis Based on Triangular Interpolations

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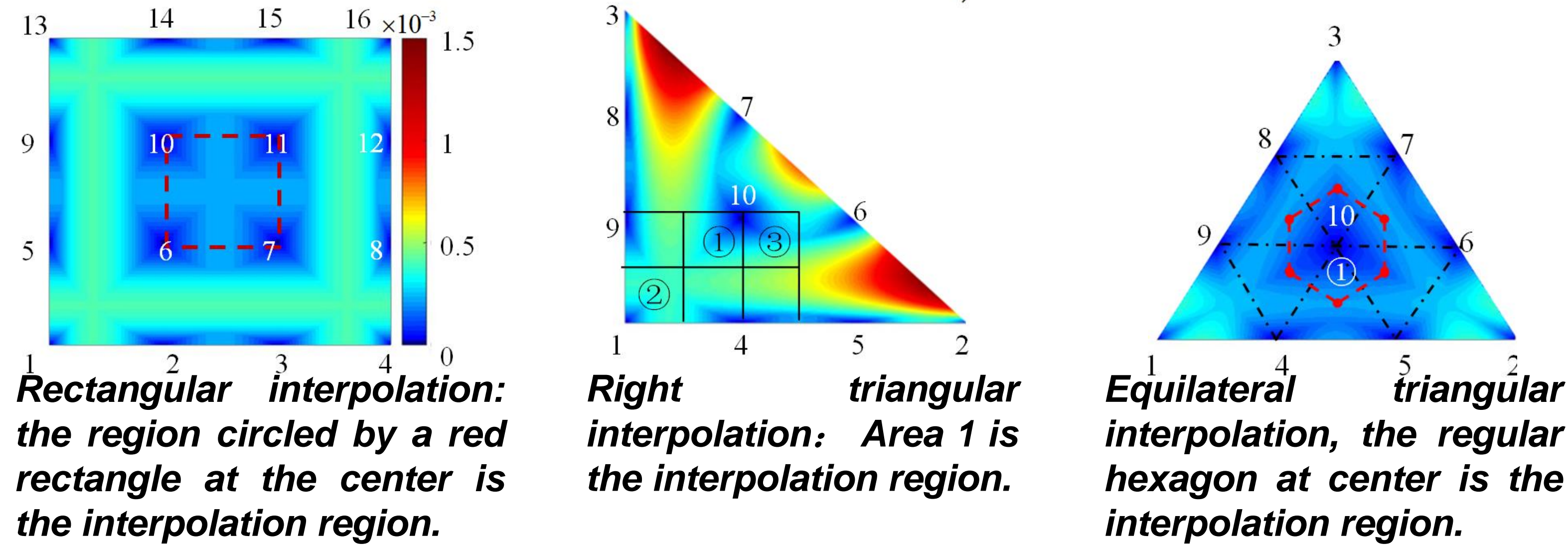
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Introduction

- Interpolation is widely used in computational electromagnetics (CEM), including finite element method (FEM) and method of moments (MoM).
- We focus on studying the high order interpolations based on the right triangular, equilateral triangular grids with the minimum interpolation errors and choosing the interpolation regions for given data points.

Interpolation Region

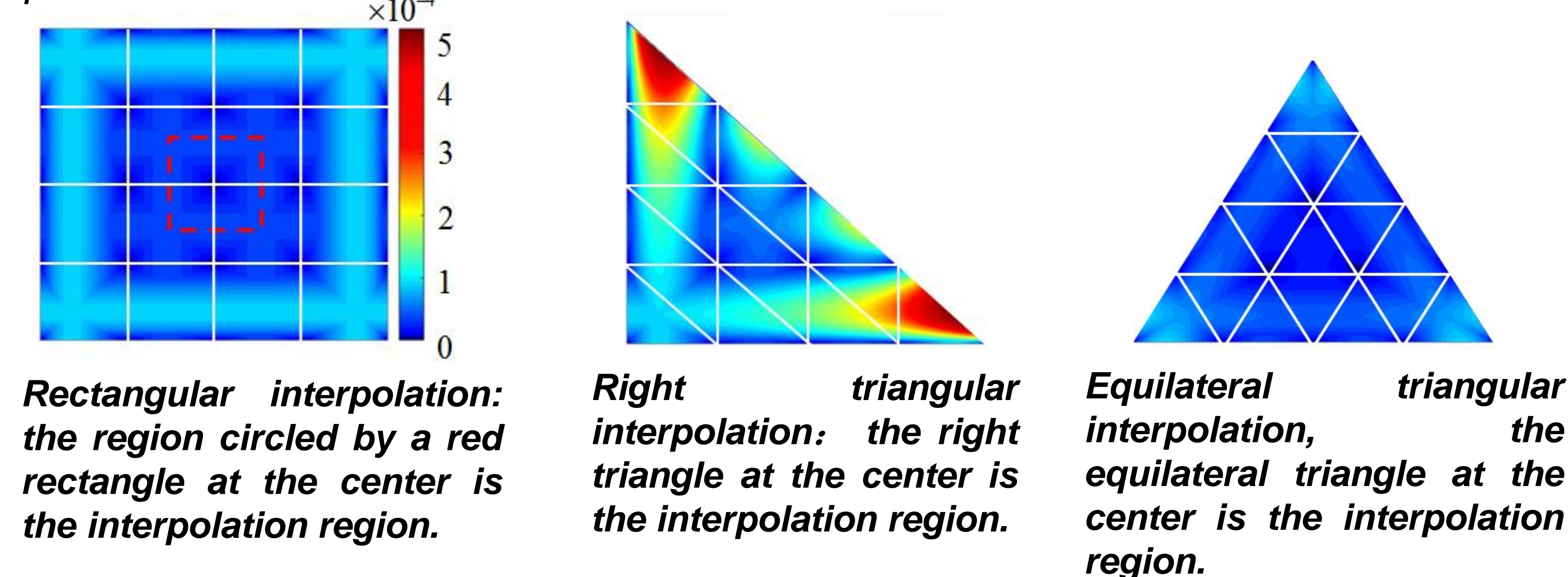
- $p=3$, maximum error distributions



The number of data points for the rectangular and triangular interpolations is **sixteen** and **ten**.

The right triangular interpolation has the **largest** errors, and the maximum errors of the right triangular interpolation and rectangular interpolation are close to each other.

- $p=4$, maximum error distributions



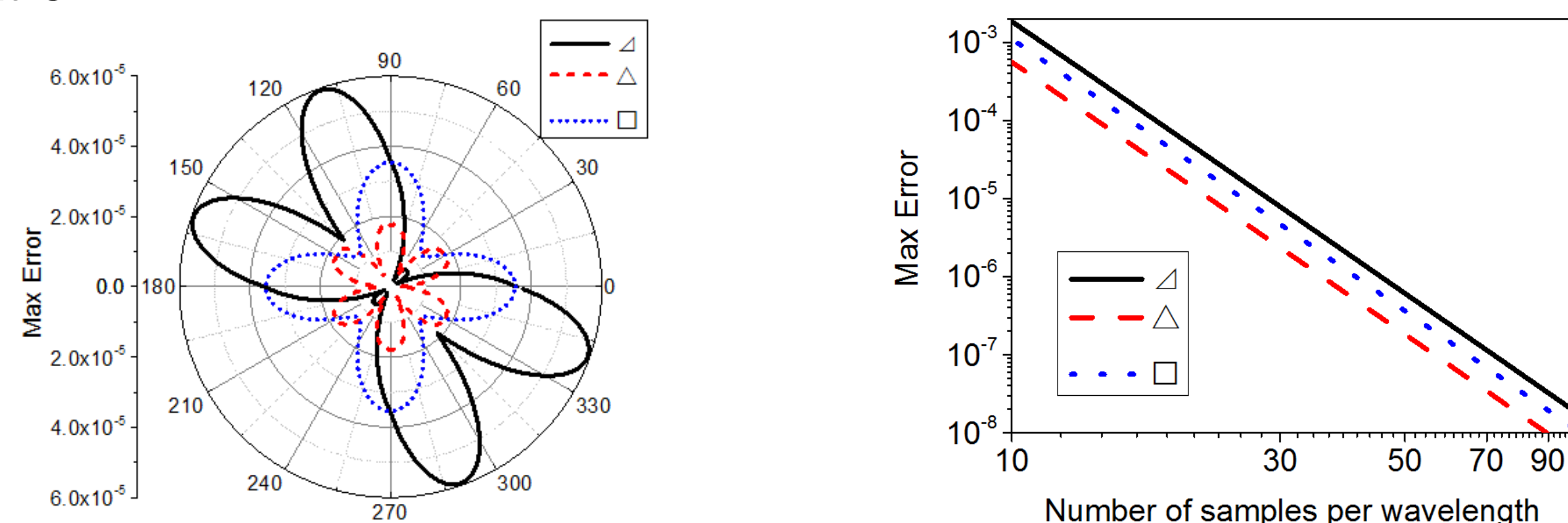
The number of data points for the rectangular and triangular interpolations is **twenty-five** and **fifteen**, respectively.

Numerical results

- $p=3$

The max error of the right triangular interpolations is **107%** more than the rectangular interpolations and the RMS error is **58.1%** more than the rectangular interpolation, but it uses **six** data points less (10 to 16).

The max error of the equilateral triangular interpolations is **same** as the rectangular interpolation, but the RMS error is **71.4%** of the rectangular interpolation.

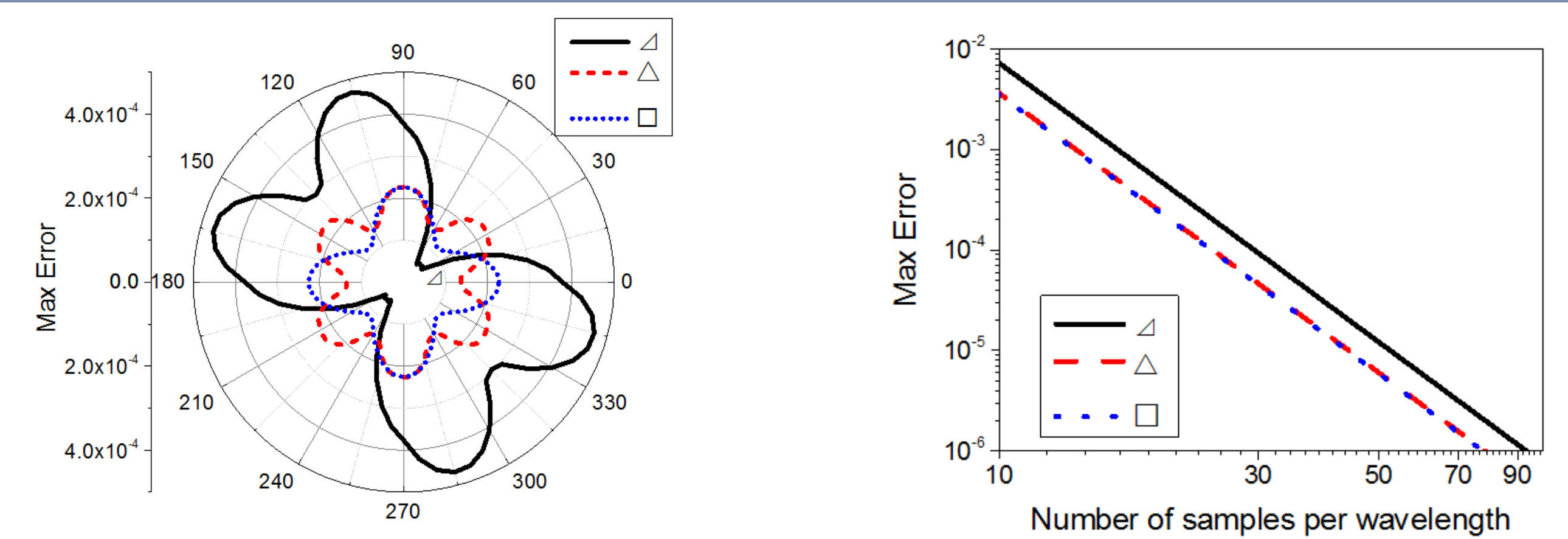


The maximum errors of the right triangular interpolation (∇), equilateral triangular interpolation (\triangle) and rectangular interpolation (\square)

- $p=4$

The max error of the right triangular interpolations is **67%** more than the rectangular interpolations and the RMS is **36.7%** more than the rectangular interpolation, but it uses ten data points less (15 to 25).

The max error of the equilateral triangular interpolations is **50.3%** of the rectangular interpolation and the RMS error is **54.4%** of the rectangular interpolation.



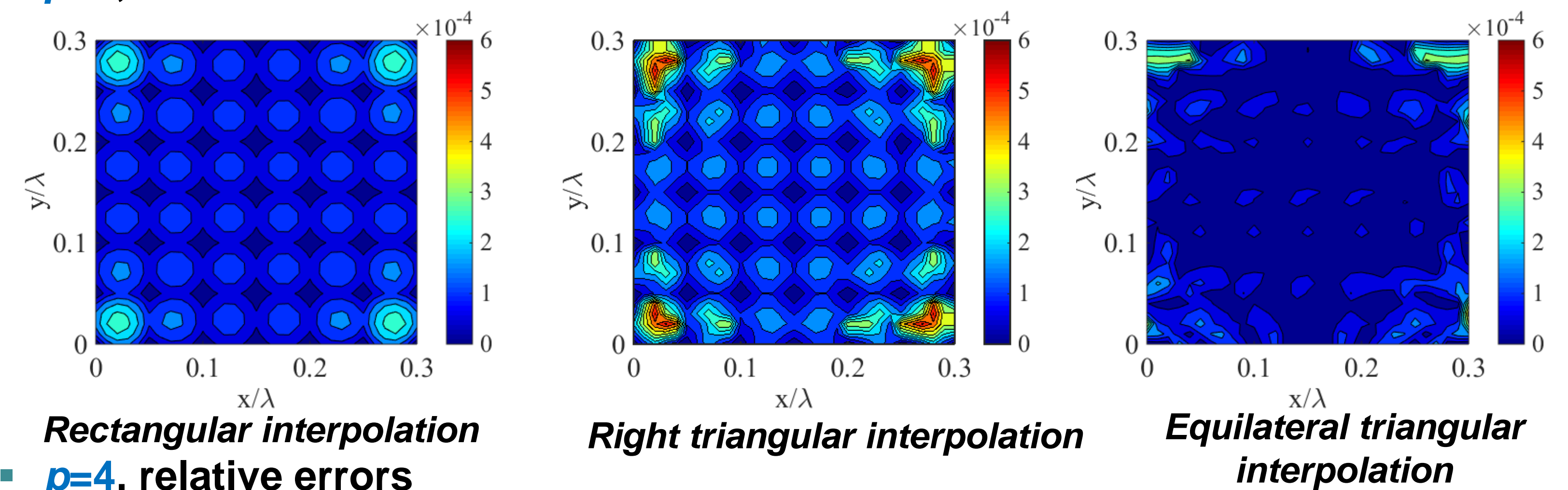
for different directions, $\lambda/h = 20$

for different λ/h

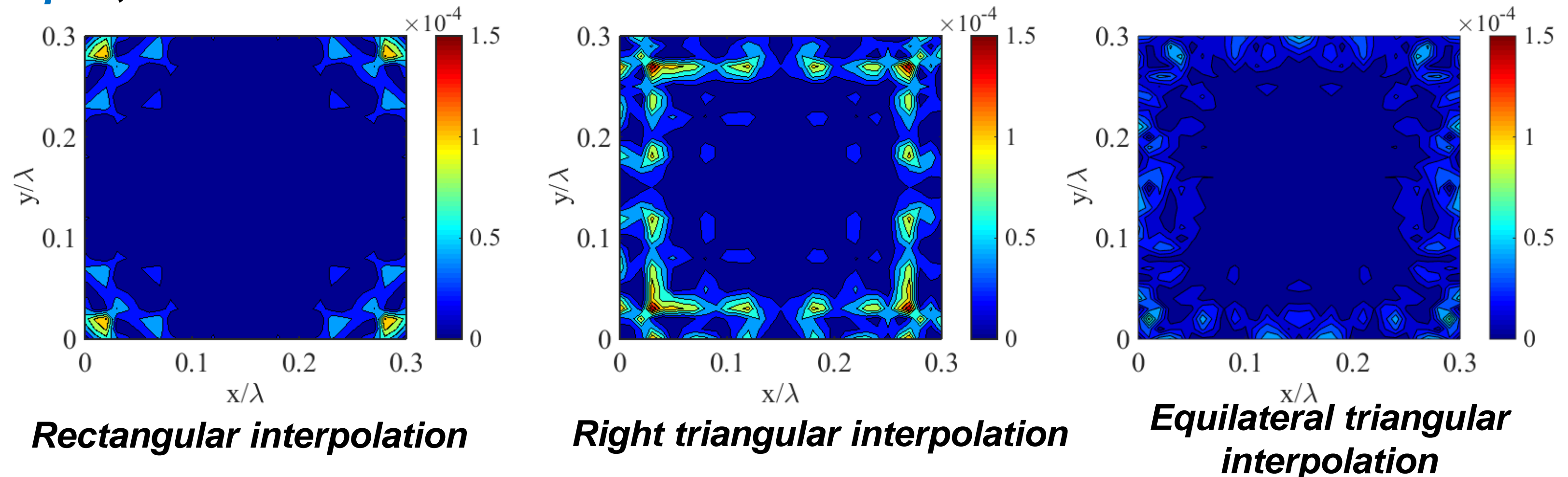
The maximum errors of the right triangular interpolation (∇), equilateral triangular interpolation (\triangle) and rectangular interpolation (\square)

Applications

- Using the Ewald-accelerated doubly PGFs as an application.
- The right triangular interpolation has **largest** interpolation error for the third and fourth order.
- The errors of equilateral triangular interpolation are **smallest**.
- The rectangular and equilateral triangular interpolation cost similar interpolation time.
- $p=3$, relative errors



- $p=4$, relative errors



Order	Interpolation time		
	\square	∇	\triangle
3	0.022s	0.015s	0.022s
4	0.036s	0.021s	0.034s

Table Interpolation time of the right triangular interpolation (∇), equilateral triangular interpolation (\triangle) and rectangular interpolation (\square)

Conclusion

- The triangular interpolations are more efficient. The number of data points for triangular interpolations is up to 50% less than the rectangular interpolations.
- The right triangular interpolation uses the same sampling grid as the rectangular interpolation
- The equilateral triangular grids have number of sampling points 15.5% more than the rectangular grids. The RMS and maximum errors are up to half of the rectangular interpolation ($p=4$).

References

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