Explicit Newmark-FDTD Method Based on Maxwell's Equation

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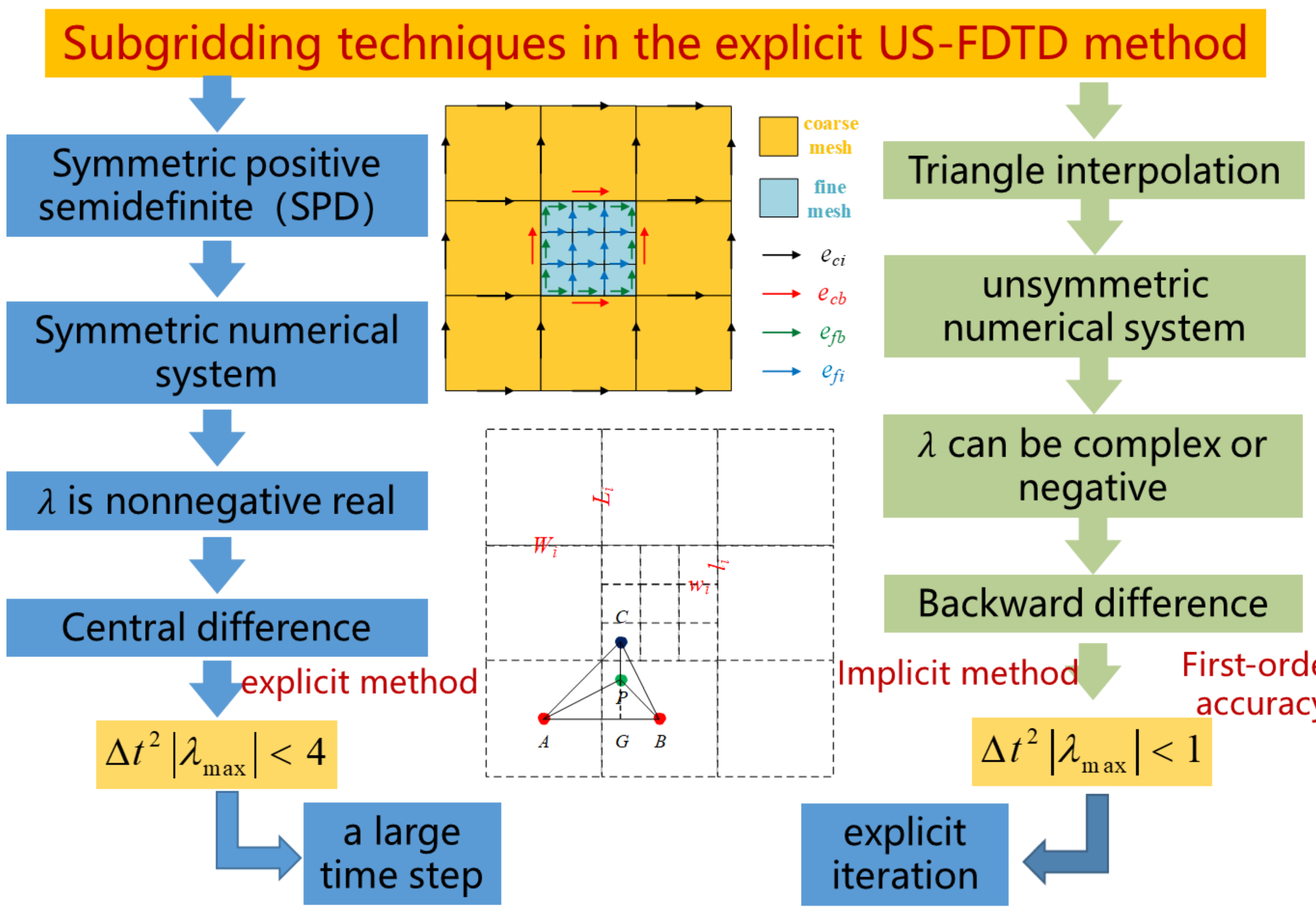
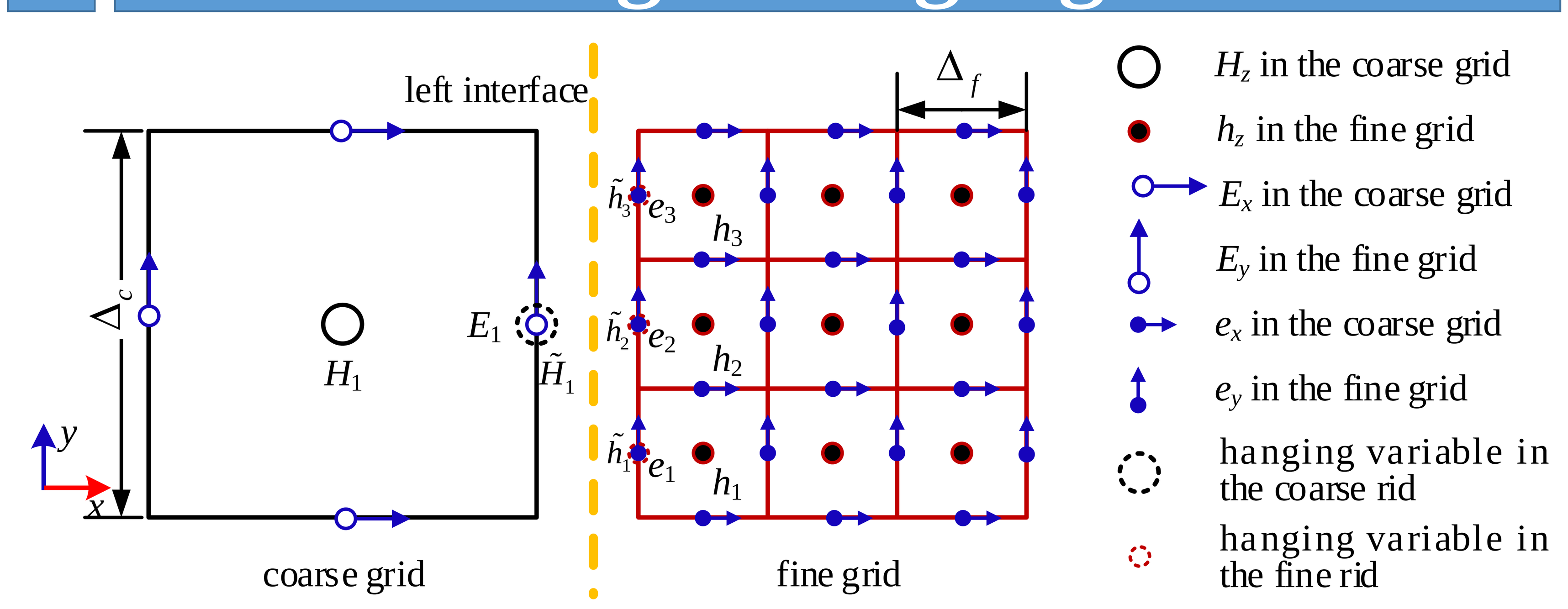
Motivation

There are two subgridding techniques in the existing explicit US-FDTD method. One is the symmetric positive semidefinite(SPD), and the other is the triangle interpolation. In this paper, Newmark method is used to discretize time-related items, and the condition for Neumann series expansion of the inverse of coefficient matrix in Newmark-FDTD is derived. Finally, the explicit iteration of Newmark-FDTD algorithm is realized.



A stable subgridding algorithm



The electric field in **the coarse grid near the interface** can be calculated from

$$\varepsilon\frac{\partial E_1}{\partial t}=\frac{H_1-\tilde{H}_1}{\Delta_c/2}$$

Similarly, we can obtain the electric fields of **fine grids near the interface** from

$$\varepsilon\frac{\partial e_n}{\partial t}=\frac{\tilde{h}_n-h_n}{\Delta_f/2}\quad (n=1,2,\ldots,r)$$

The coarse and fine grids are connected by **imposing a suitable relation** between the electric and magnetic fields at the interface.

$$e_1=e_2=\ldots=e_r=E_1\qquad,\qquad \tilde{H}_1=\frac{1}{r}\left(\tilde{h}_1+\tilde{h}_2+\ldots+\tilde{h}_n\right)$$

According to above formulas, **the hanging variables can be eliminated** and we can obtain

$$\varepsilon\frac{\partial E_1}{\partial t}=\frac{H_1-\left(h_1+h_2+\ldots+h_n\right)/r}{\Delta_c/2/\left(1+1/r\right)}\quad (n=1,2,\ldots,r)$$

Newmark-FDTD method based on Maxwell's equations

Standard form

$$\nabla\times\boldsymbol{H}=\varepsilon\frac{\partial\boldsymbol{E}}{\partial t}\qquad \nabla\times\boldsymbol{E}=-\mu\frac{\partial\boldsymbol{H}}{\partial t}$$

Matrix form

$$\boldsymbol{S}_h\{h\}=\boldsymbol{D}_\varepsilon\frac{\partial\{e\}}{\partial t}\qquad \boldsymbol{S}_e\{e\}=-\boldsymbol{D}_\mu\frac{\partial\{h\}}{\partial t}$$

Newmark discretization（$\beta=0.25$，$\gamma=0.5$）

The iteration equation of the magnetic field — **implicit iteration**

$$\left(\boldsymbol{I}+0.25\Delta t^2\cdot\boldsymbol{S}\right)\{h\}^{n+1}=-0.5\Delta t^2\cdot\boldsymbol{S}\{h\}^n+\left(\boldsymbol{I}-0.25\Delta t^2\cdot\boldsymbol{S}\right)\{h\}^{n-1}-\Delta t\cdot\boldsymbol{D}_{1/\mu}\boldsymbol{S}_e\{e\}^n-\Delta t\cdot\boldsymbol{D}_{1/\mu}\boldsymbol{S}_e\{e\}^{n-1}$$

The iteration equation of the electric field — **explicit iteration**

$$\{e\}^{n+1}=\{e\}^{n-1}+0.5\Delta t\cdot\boldsymbol{D}_{1/\varepsilon}\boldsymbol{S}_h\{h\}^{n+1}+\Delta t\cdot\boldsymbol{D}_{1/\varepsilon}\boldsymbol{S}_h\{h\}^n+0.5\Delta t\cdot\boldsymbol{D}_{1/\varepsilon}\boldsymbol{S}_h\{h\}^{n-1}$$

Explicit solution of Newmark-FDTD

if $\rho(0.25\Delta t2\,\boldsymbol{S})<1$, then

$$(\boldsymbol{I}+0.25\Delta t^2\boldsymbol{S})^{-1}=\sum_{k=0}^{\infty}\left(-0.25\Delta t^2\boldsymbol{S}\right)^k$$

$$\rho(0.25\Delta t^2\boldsymbol{S})<1\ \Leftrightarrow\ 0.25\Delta t^2\left|\lambda_{\max}\right|<1\ \Leftrightarrow\ 0.25\Delta t^2\left|\lambda_{\max}\right|<1$$

solution frequency → time step → find eigenvectors $\boldsymbol{V}_h$

$\Delta t^2\lambda>4$ ↓ $\boldsymbol{S}_l=\boldsymbol{S}-\boldsymbol{V}_h\boldsymbol{V}_h^T\boldsymbol{S}$

eliminate these eigenvectors → $\rho(0.25\Delta t^2\boldsymbol{S}_l)<1$

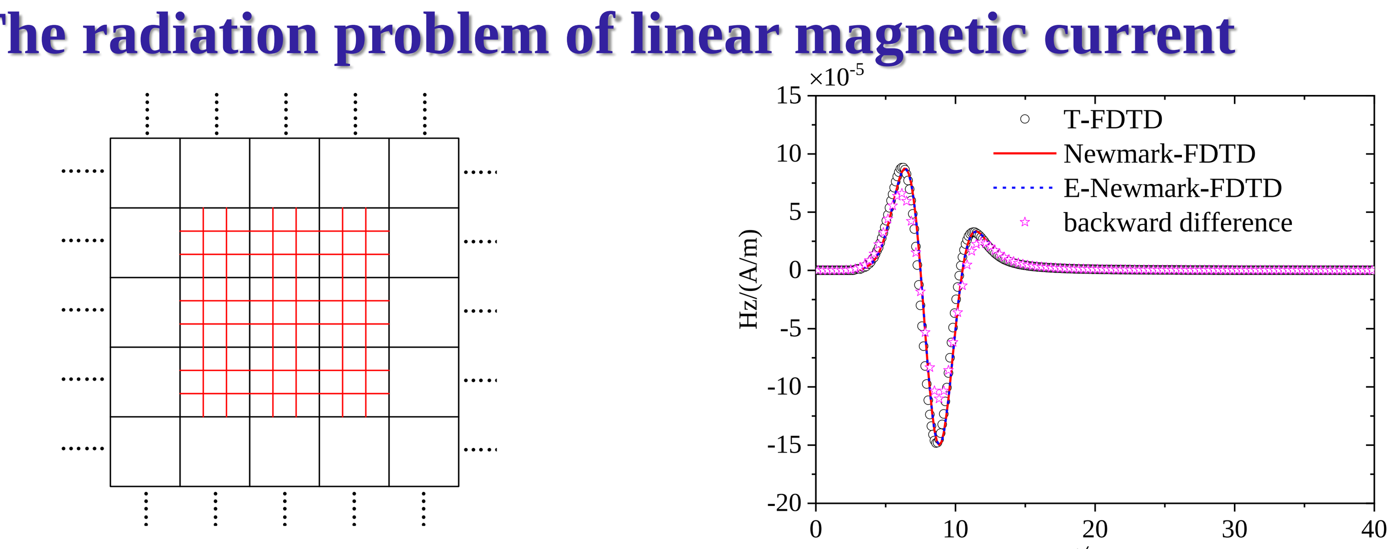
$$\left(\boldsymbol{I}+0.25\Delta t^2\boldsymbol{S}_l\right)^{-1}=\boldsymbol{I}-0.25\Delta t^2\boldsymbol{S}_l+(0.25\Delta t^2\boldsymbol{S}_l)^2-(0.25\Delta t^2\boldsymbol{S}_l)^3+\cdots+(-1)^k(0.25\Delta t^2\boldsymbol{S}_l)^k$$

↓ **matrix-vector multiplication**

$$\begin{aligned}&\left(\boldsymbol{I}+0.25\Delta t^2\boldsymbol{S}_l\right)^{-1}\{\Phi\}\\&=\left[\boldsymbol{I}-0.25\Delta t^2\boldsymbol{S}_l+(0.25\Delta t^2\boldsymbol{S}_l)^2-(0.25\Delta t^2\boldsymbol{S}_l)^3+\cdots+(0.25\Delta t^2\boldsymbol{S}_l)^n\right]\{\Phi\}\\&=\boldsymbol{I}\{\Phi\}-0.25\Delta t^2\boldsymbol{S}_l\{\Phi\}+(0.25\Delta t^2\boldsymbol{S}_l)^2\{\Phi\}-(0.25\Delta t^2\boldsymbol{S}_l)^3\{\Phi\}+\cdots+(-1)^n(0.25\Delta t^2\boldsymbol{S}_l)^n\{\Phi\}\end{aligned}$$

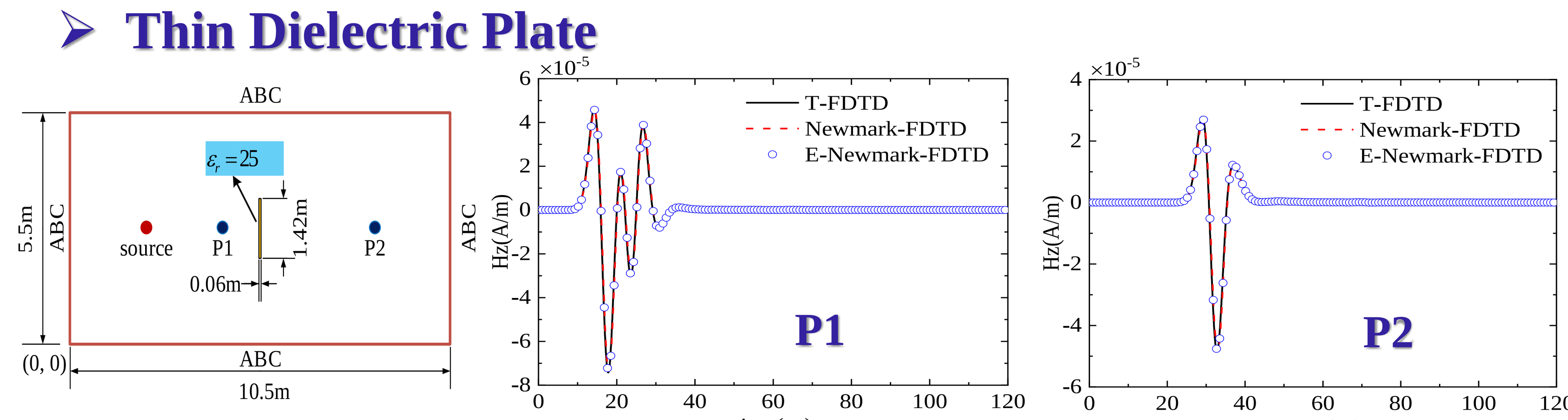
Numerical examples

➢ **The radiation problem of linear magnetic current**



Methods	T- FDTD	Newmark-FDTD	E-Newmark-FDTD	BD
Mesh	fine mesh	subgridding	subgridding	subgridding
Time step(s)				
EVD time(s)	-	-	17.89	-
Iteration time(s)	29.41	39.86	3.79	35.94
Total time(s)	29.41	39.86	21.68	35.94

➢ **Thin Dielectric Plate**



Methods	T-FDTD	Newmark-FDTD	E-Newmark-FDTD
Mesh		subgridding	
Time(s)	2.3	17.1	1.1

Conclusion

We have proposed an explicit unconditionally stable Newmark-FDTD algorithm with subgridding technique to solve the electromagnetic problems. It has good accuracy and calculation efficiency.

References

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