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## Abstract

In this paper, A weighted Domain Decomposition Method (Weighted DDM) enhanced by the hybrid fast algorithm, i.e. the multilevel accelerated Cartesian expansion-multilevel fast multipole algorithm (MLACE-MLFMA), is adopted to analyze the electromagnetic scattering from large metallic targets. In the computational scheme, the surface integral equation (SIE) is adopted and the MLACE-MLFMA is utilized to accelerate the matrix-vector multiplications (MVMs). Compared with the unenhanced weighted DDM, the computational efficiency has been largely improved which can be demonstrated by the numerical results.

## Introduction

In the practical simulations, scatterings of the large target are always been considered. However, considering the memory consumption and computational complexity of Method of Moment (MoM), high costs for calculating scatterings of large targets are inevitable. In [1]-[3], significant fast algorithms to accelerate iterations are proposed, which are the multilevel fast multipole algorithm (MLFMA), the multilevel accelerated Cartesian expansion algorithm (MLACEA) and the multilevel accelerated Cartesian expansion-multilevel fast multipole algorithm (MLACE-MLFMA). By applying the MLFMA or MLACE-MLFMA, the memory cost and computational complexity can be reduced. However, preconditioners, resulting in more memory and computation costs, may be needed to improve the convergence of the iterative solution.

To improve the iterative convergence and avoiding extra memory consumption, domain decomposition method (DDM) is proposed and developed. In the DDM, transmission conditions will be constructed to ensure the continuity of current vector between artificial boundary metallic edges. Here, overlapping subdomains [5] or meshed unknowns on artificial edges [6] can be utilized as transition zone. In [5], [6], the Gauss-Seidel-like iterative scheme is utilized to solve the DDM equation.

Recently, a weighted domain decomposition method (weighted DDM) [7] is proposed for scattering problems by using the Surface Integral equation (SIE). In the weighted DDM, the solution process has been divided into three main steps: calculation of subdomain scatterings, solution of the weighted coefficients and judgment of the iterative convergence. Moreover, Far Field Approximation (FFA) [8] is used to accelerate the computation of interactions between subdomains.

## Theory

$$\mathbf{Z}_{ii} \mathbf{I}_i^k = \begin{cases} \mathbf{V}_i & , k=1 \\ \mathbf{V}_i^k & , k>1 \end{cases}$$

LU decomposition method

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{V} \quad (1)$$

$$\mathbf{I} = \sum_{k=1}^M \beta_{i,k} \mathbf{I}_{i,k} \quad (2)$$

$$\sum_{k=1}^M \beta_{i,k} \mathbf{Z} \cdot \mathbf{I}_{i,k} = \mathbf{V} \quad (3)$$

In order to calculate  $\beta_{i,k}$ ,  $\mathbf{Z} \cdot \mathbf{I}_{i,k}$  is considered as the test function.

$$\mathbf{Z}'_{Mn \times Mn} \beta'_{Mn \times 1} = \mathbf{V}'_{Mn \times 1} \quad (4)$$

$$\mathbf{Z}'(ik) = (\mathbf{Z} \cdot \mathbf{I}_{i,k})^H \cdot (\mathbf{Z} \cdot \mathbf{I}_{i,k}) \quad (5)$$

$$\mathbf{V}'(ik) = (\mathbf{Z} \cdot \mathbf{I}_{i,k})^H \cdot \mathbf{V} \quad (6)$$

(4) is solved by the LU decomposition direct solver.

$$error(k) = \frac{\|\mathbf{V} - \mathbf{V}^k\|_2}{\|\mathbf{V}\|_2}$$

$$\mathbf{V}_i^k = \mathbf{V}_i - (\mathbf{Z} \cdot \mathbf{I}^{k-1})_i = \mathbf{V}_i - \sum_{j=1}^{k-1} \beta_{i,j} \mathbf{Z} \cdot \mathbf{I}_{i,j}$$

1<sup>st</sup> step

2<sup>nd</sup> step

3<sup>rd</sup> step

## Memory and computational costs

Total memory cost:

$$T_{memory}^{total} = p \left( \frac{N}{M} \right)^2 + MnN$$

Total computational cost:

$$T_{operator}^{total} = nN^2 + (Mn)^2 N + M \left( \frac{N}{M} \right)^3 + nM \left( \frac{N}{M} \right)^2 + T_{repeated}$$

MoM based weighted DDM

The computational cost for calculating

mutual-coupling matrices:  $T_{repeated}$

Total memory cost:

$$T_{memory}^{total} = N \log N + p \left( \frac{N}{M} \right)^2 + MnN$$

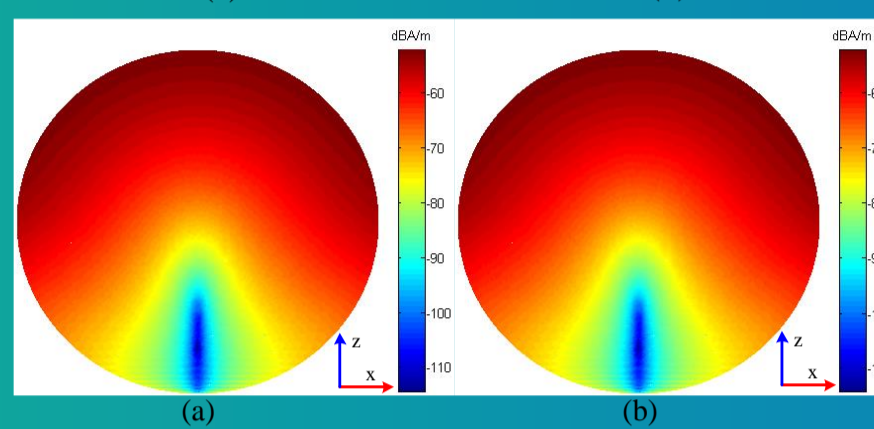
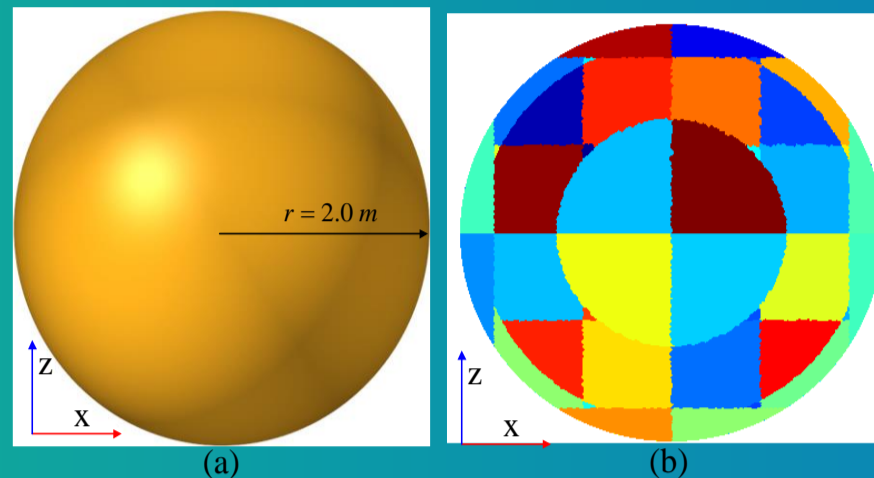
MLACE-MLFMA based weighted DDM

Total computational cost:

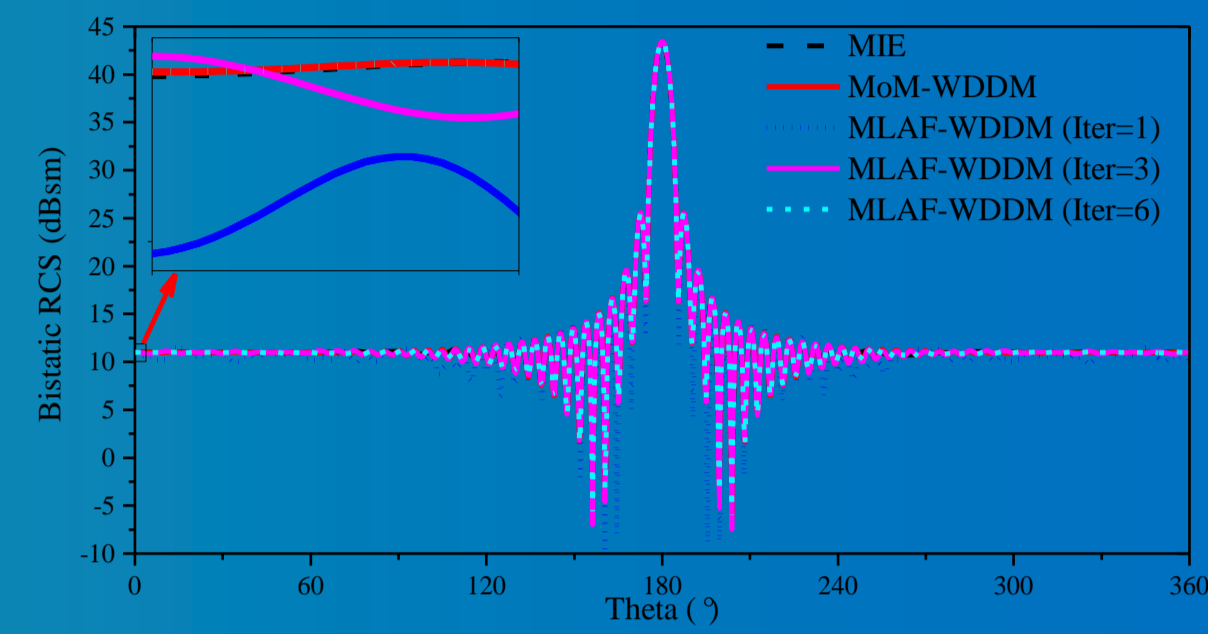
$$T_{operator}^{total} = MnN \log N + (Mn)^2 N + M \left( \frac{N}{M} \right)^3 + nM \left( \frac{N}{M} \right)^2$$

## Numerical results

### A. Metallic sphere conductor



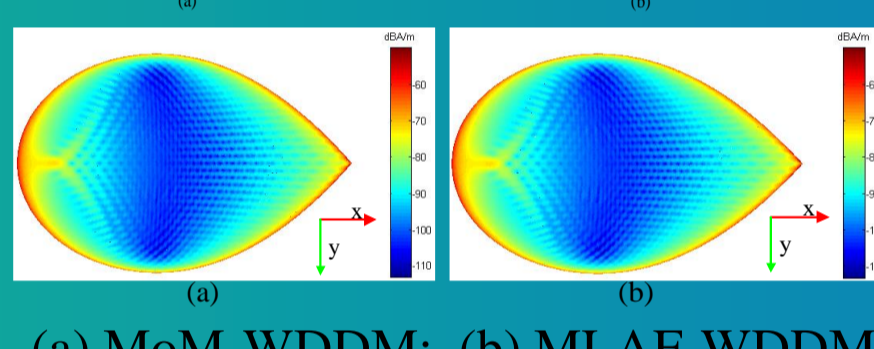
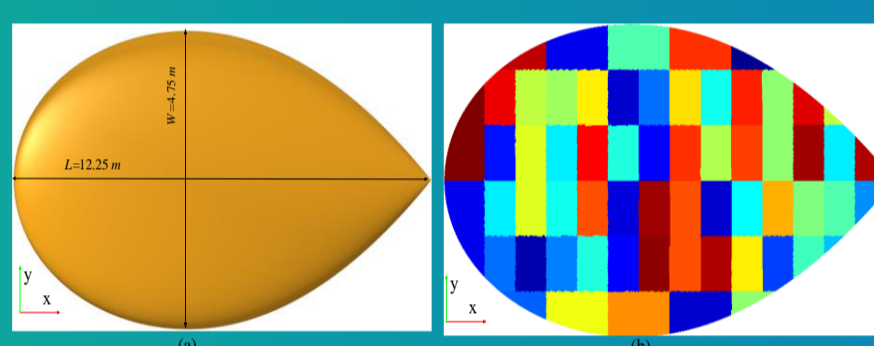
(a) MoM-WDDM; (b) MLAF-WDDM



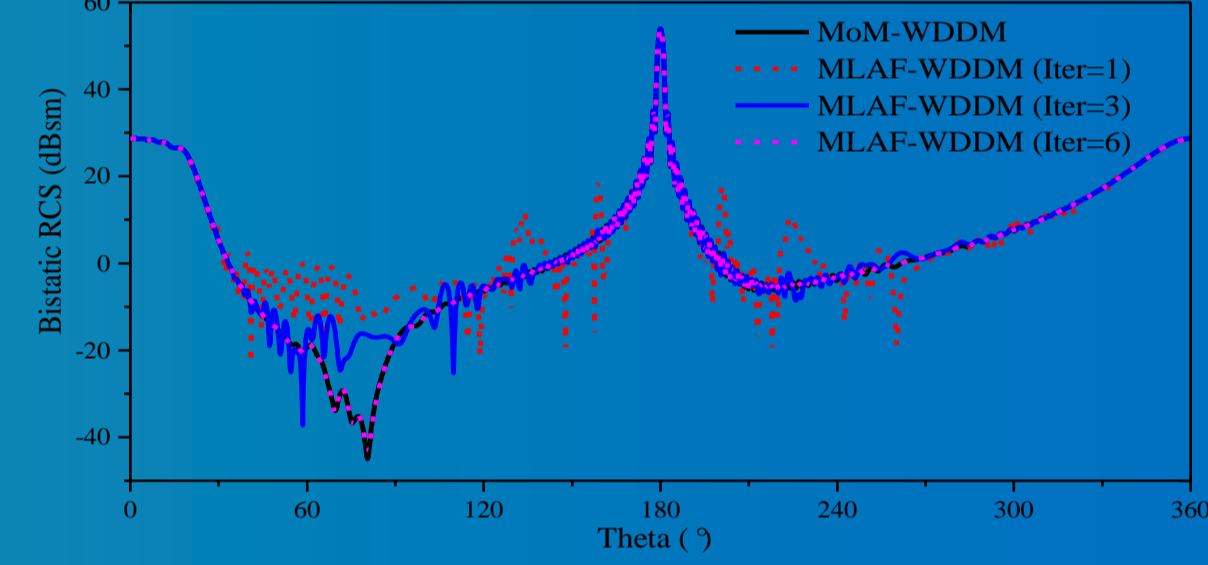
“MLAF”=MLACE-MLFMA

Frequency: 1.0 GHz  
Triangles: 99730  
Unknowns: 149595  
Subdomains: 80

### B. NASA almond

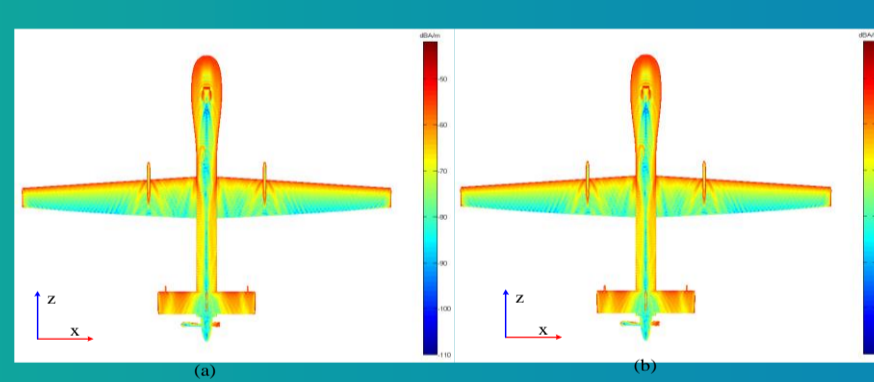
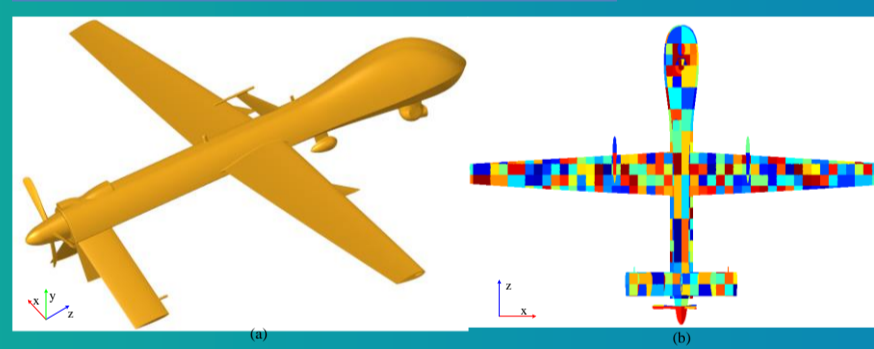


(a) MoM-WDDM; (b) MLAF-WDDM



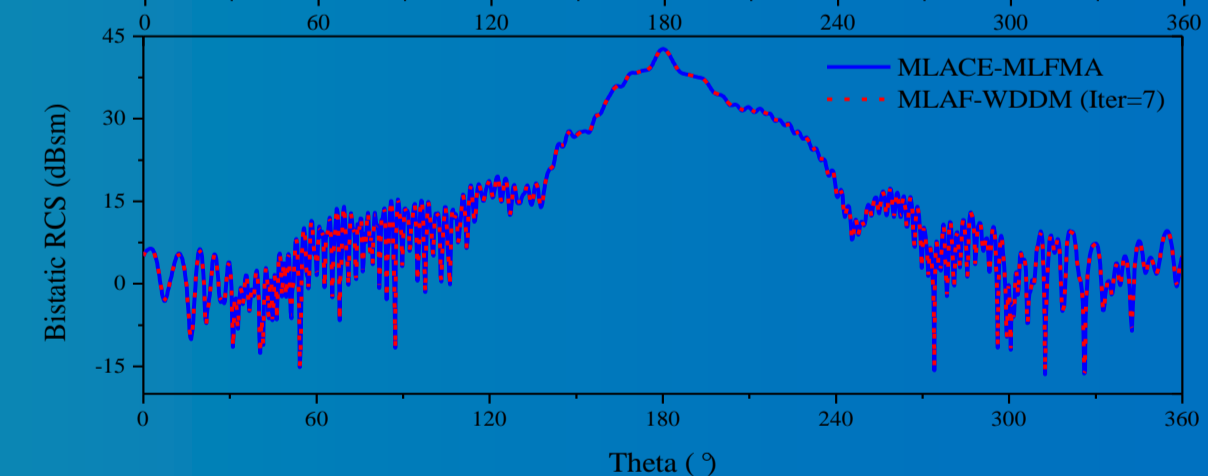
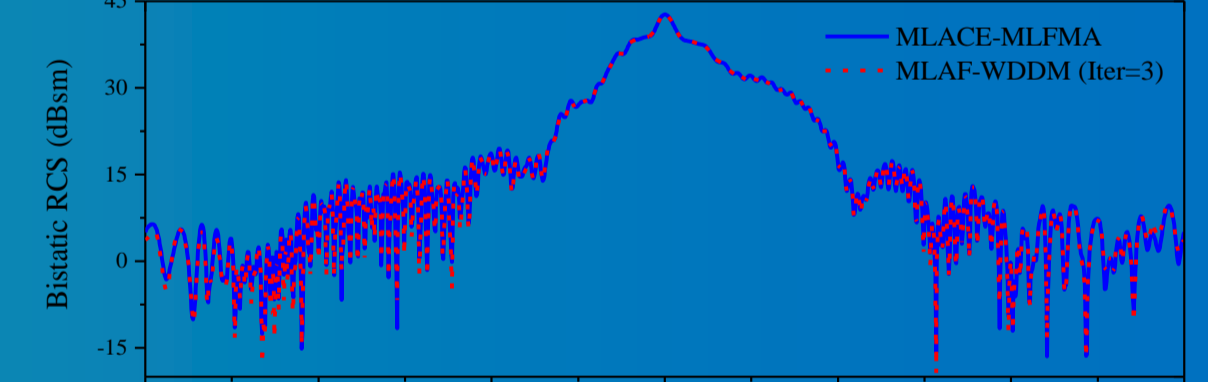
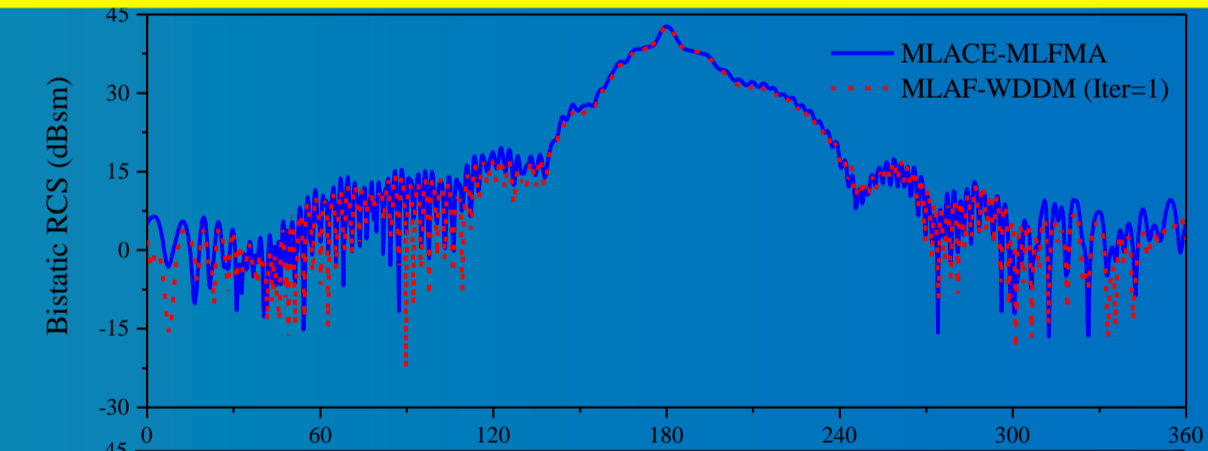
Frequency: 1.0 GHz  
Triangles: 210818  
Unknowns: 316227  
Subdomains: 124

### C. Aircraft



(a) MLACE-MLFMA; (b) MLAF-WDDM

Frequency: 2.0 GHz  
Triangles: 533418  
Unknowns: 800127  
Subdomains: 448



Models	Peak memory (GB)		Iterations		Time cost (minutes)	
	MoM-WDDM	MLAF-WDDM	MoM-WDDM	MLAF-WDDM	MoM-WDDM	MLAF-WDDM
Sphere	11.26	13.33	6	6	196.99	16.84
Almond	18.62	22.85	6	6	857.34	36.10
Aircraft	--	38.07	--	7	--	188.30

## Conclusion

In our work, the weighted DDM is accelerated by the multilevel accelerated Cartesian expansion-multilevel fast multipole algorithm (MLACE-MLFMA). Here, the MLACE-MLFMA is mainly utilized to accelerate the matrix-vector multiplication. The validity of the accelerated weighted DDM has been demonstrated by the proposed simulations. Compared with the unenhanced weighted DDM, computational efficiency has been highly improved despite of the increase of memory. In addition, the size of the subdomains is not required to be uniform, which increases the domain decomposition flexibility.

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